Phones must be turned OFF and put away. No scratch paper. No graphing calculator. All of your solutions must be on this test paper. No credit will be given for solutions if work is not shown. I expect clear presentations with words of explanation.

$\langle -4,-2,3\rangle \quad \langle 7,-2,1\rangle$

(1) Given the vectors $\mathbf{a} = -4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = 7\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and find the following: (4 points each)

a) $\mathbf{a} \times \mathbf{b}$ $\boxed{\langle 4, 25, 22\rangle}$

$$\begin{vmatrix} i & j & k \\ -4 & -2 & 3 \\ 7 & -2 & 1 \end{vmatrix}$$

b) the angle between a and b $\boxed{\theta = \cos^{-1}\left(\frac{-21}{\sqrt{29}\sqrt{54}}\right) \approx 122^\circ}$

$$\cos\theta = \frac{a \cdot b}{\|a\|\|b\|} = \frac{-21}{\sqrt{29}\sqrt{54}}$$

c) $\text{proj}_a \mathbf{b}$ $\boxed{\left\langle \frac{84}{29}, \frac{42}{29}, \frac{-63}{29} \right\rangle}$

$$\frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a} = \frac{-21}{29}\vec{a}$$

d) a vector of length 3 in the direction of **b** $\boxed{\left\langle \frac{7}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle}$

$$\vec{u} = \frac{1}{\|\vec{b}\|}\vec{b} = \frac{1}{\sqrt{54}}\vec{b} \quad \text{So } 3u = \frac{3}{\sqrt{54}}\vec{b} = \frac{3}{3\sqrt{6}}\vec{b} = \frac{1}{\sqrt{6}}\vec{b}$$

e) a value for k such that < k, 8,-6 > is orthogonal **b** $\boxed{K = \frac{22}{7}}$

7 -2 1

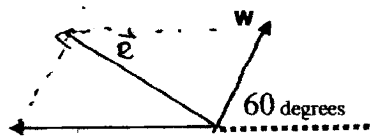
dot product zero

$$7k - 16 - 6 = 0$$
$$7k = 22$$

f) If point P is (1,9,1) and point Q is (0, 10, 4) is $\overrightarrow{PQ}$ parallel to a? $\boxed{\text{NO}}$

$PQ = \langle -1, 1, 3\rangle$

(2) Given the forces **v** and **w** as shown, where $\|\vec{v}\| = 40$ lbs and $\|\vec{w}\| = 20$ lbs, find the resultant (9 points



v

Find components

$$\vec{w} = \langle 20\cos 60^\circ, 20\sin 60^\circ\rangle = \langle 10, 10\sqrt{3}\rangle$$
$$\vec{v} = \langle -40, 0\rangle$$
$$\vec{R} = \langle -30, 10\sqrt{3}\rangle$$

$$\|\vec{R}\| = \sqrt{900+300} = \sqrt{1200} = 20\sqrt{3} \text{ lbs}$$

$$\tan\theta = \frac{10\sqrt{3}}{-30} = -\frac{\sqrt{3}}{3} \quad \theta = 150^\circ$$

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(3) Find the intersection point of the following lines, if any, and find the equation of the plane containing them.

$$L_1 \begin{cases} x = 2t-1 \\ y = 1-t \\ z = 3t \end{cases} \qquad L_2 \begin{cases} x = 1+s \\ y = 2s \\ z = 3-2s \end{cases}$$

$\vec{v}_1 = \langle 2,-1,3\rangle$ ~~(13 points)~~ $\vec{v}_2 = \langle 1,2,-2\rangle$

(7 points)

Intersect?
$$\begin{cases} 2t-1 = 1+s \\ 1-t = 2s \\ 3t = 3-2s \end{cases} \Rightarrow \begin{cases} s = 2t-2 \\ 1-t = 2s \\ 3t = 3-2s \end{cases}$$
$1-t = 2(2t-2) \Rightarrow t=1, s=0$ satisfies first two

$3t = 3-2s \leftarrow$ must show check in third eqn. $3(1) = 3-2(0)$? ✓

So intersects at $\boxed{(1,0,3)}$

For plane need point: $P(1,0,3)$

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 1 & 2 & -2 \end{vmatrix} = \langle -4,7,5\rangle$$

plane: $-4(x-1) + 7y + 5(z-3) = 0$

or $-4x+7y+5z = 11$ can check!!

(4) Prove: If **a** and **b** are vectors in R^3 and c is a real number then $(c\mathbf{a})\cdot\mathbf{b} = c(\mathbf{a}\cdot\mathbf{b})$..
(6 points)

Proof: Let $\vec{a} = \langle a_1, a_2, a_3\rangle$ and $\vec{b} = \langle b_1, b_2, b_3\rangle$, so $c\vec{a} = \langle ca_1, ca_2, ca_3\rangle$

Then $(c\vec{a})\cdot\vec{b} = \langle ca_1, ca_2, ca_3\rangle \cdot \langle b_1, b_2, b_3\rangle$

$= (ca_1)b_1 + (ca_2)b_2 + (ca_3)b_3$

$= c(a_1b_1 + a_2b_2 + a_3b_3)$

$= c(\vec{a}\cdot\vec{b})$ ∎

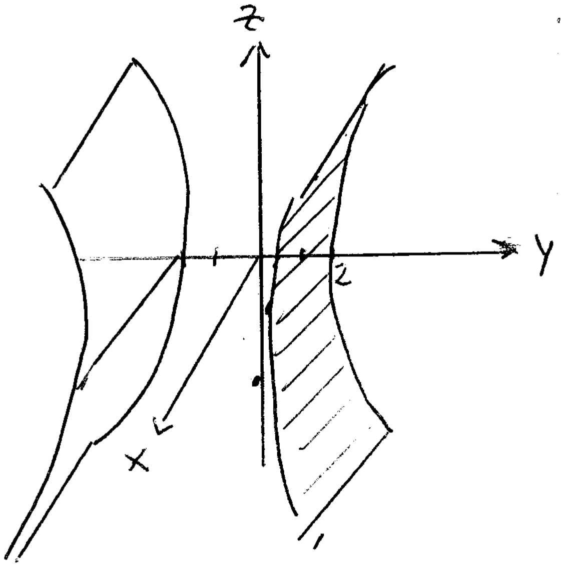
(5) On separate axes, sketch a graph of the following surfaces. Name the surface and give pertinent information such as traces. (21 points)

(a) $9y^2 - 4z^2 = 36$

(b) $9x^2 + y^2 - z^2 = 9$

(c) $y = \sqrt{4x^2 + z^2}$

a) No x $\Rightarrow$ Cylinder



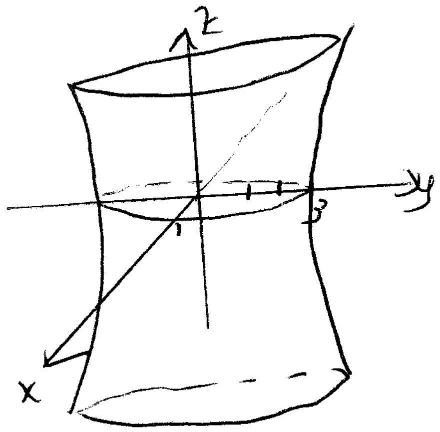
b) Hyperbolad One Sheet

if $z=0$ $\quad 9x^2+y^2=9$

$x^2+\frac{y^2}{9}=1$

$z=3$ $\quad 9x^2+y^2=18$

$\frac{x^2}{2}+\frac{y^2}{18}=1$

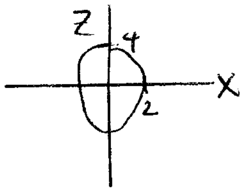


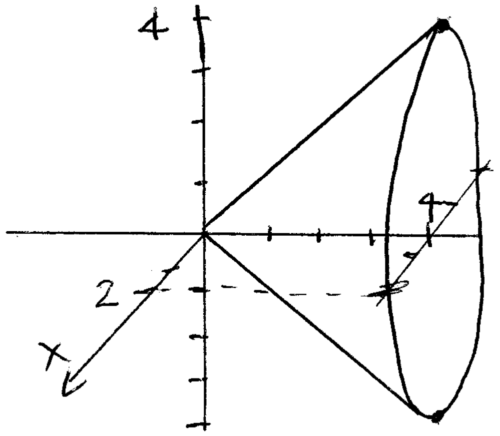
c) $y=\sqrt{4x^2+z^2}$ cone

If $y=4$ $\quad 4=\sqrt{4x^2+z^2}$

$16=4x^2+z^2$

$1=\frac{x^2}{4}+\frac{z^2}{16}$





longer in z direction

(6) Consider the following lines. Show whether they intersect, are parallel, or are skew. IF they intersect, find the point of intersection AND find the equation of the plane containing the lines. IF they are parallel or skew, find the distance between them.

(16 points)

$$L_1 \begin{cases} x = 2t+1 \\ y = t \\ z = 4t+1 \end{cases} \qquad L_2 \begin{cases} x = s \\ y = 2s-2 \\ z = 3s-2 \end{cases}$$

$P_1(1,0,1)$ $\quad$ $P_2(0,-2,-2)$

$\vec{V}_1 = \langle 2,1,4 \rangle$ $\quad$ $\vec{V}_2 = \langle 1,2,3 \rangle$

The lines are not parallel since $\vec{V}_1 \neq c\vec{V}_2$.

Intersect?

$$\begin{cases} 2t+1 = s \\ t = 2s-2 \\ 4t+1 = 3s-2 \end{cases} \quad t = 2(2t+1)-2 \quad \left.\begin{matrix} t=0 \\ s=1 \end{matrix}\right\} \text{satisfies first two}$$

Check in third eqn.

$4(0)+1 = 3(1)-2$?

yes - intersect

So find point of intersection $(1,0,1)$

and plane: pt $(1,0,1)$

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & 4 \\ 1 & 2 & 3 \end{vmatrix} = \langle -5,-2,3 \rangle$$

plane $\quad -5(x-1)-2y+3(z-1) = 0$

or $\quad -5x-2y+3z+2 = 0$

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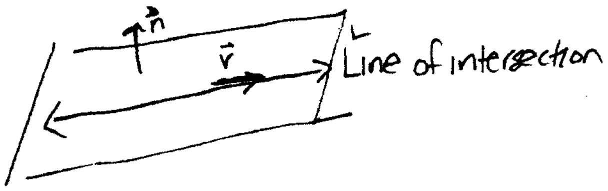
(7) Find an equation of the plane that contains the line of intersection of the planes x-z=1 and y+2z=3 and is perpendicular to the plane x+y-2z = 1 (5 points)



First Find line of intersection

$$\begin{matrix} x - z = 1 \\ y+2z = 3 \end{matrix} \Rightarrow L \begin{cases} x = t+1 \\ y = 3-2t \\ z = t \end{cases}$$

Plane: point - any point on line of intersection $(1,3,0)$

normal: $\vec{n}$ orthog to normal vector of $x+y-2z=1$ and orthog to direction vector of line of intersection

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix} = \langle -3,-3,-3 \rangle \quad \text{Use } -\tfrac{1}{3}\vec{n} = \langle 1,1,1 \rangle$$

Plane: $x-1+y-3+z=0 \qquad x+y+z=4$

(8) Find the point of intersection, if any, of the helix $\mathbf{r}_1(t) = \langle \cos t, \sin t, t\rangle$ and the curve $\mathbf{r}_2(t) = \langle 1+t, t^2, t^3\rangle$. Find the equations of the tangent lines to each of the curves at this point.

if $\vec{r}_1(t) = \vec{r}_2(s)$ for some s, t, curves intersect

Curves intersect:

$$\left.\begin{aligned} \cos t &= 1+s \\ \sin t &= s^2 \end{aligned}\right\} \quad \begin{aligned} \cos^2 t &= (1+s)^2 \\ \sin^2 t &= s^4 \end{aligned} \xrightarrow{\text{Add}} 1 = (1+s)^2 + s^4$$

$t = s^3$

$s^4 + s^2 + 2s = 0$

$s(s^3 + s + 2) = 0$

$\Rightarrow s = 0, s = -1$

so $t = 0, -1$ does not work

$\boxed{(1,0,0)}$

Tangent lines at $(1,0,0)$ $t=0$

direction vectors

$\vec{r}_1'(t) = \langle -\sin t, \cos t, 1\rangle$ $\qquad$ $\vec{r}_2'(t) = \langle 1, 2t, 3t^2\rangle$

$r_1'(0) = \langle 0, 1, 1\rangle$ $\qquad$ $r_2'(0) = \langle 1, 0, 0\rangle$

$$L_1 \begin{cases} x = 1 \\ y = t \\ z = t \end{cases} \qquad L_2 \begin{cases} x = 1+t \\ y = 0 \\ z = 0 \end{cases}$$

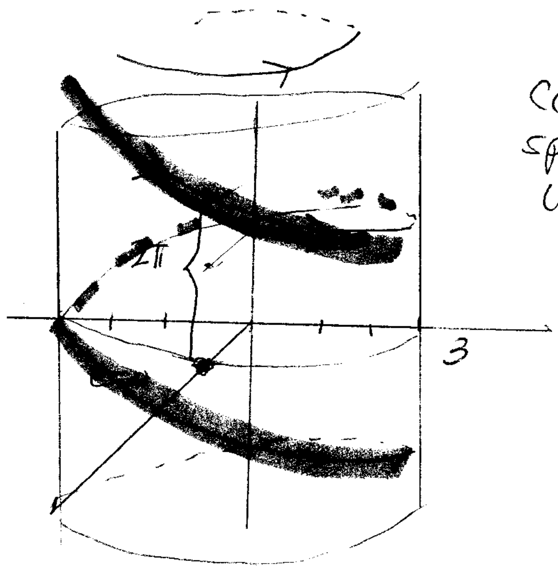
(9) Sketch the graph of $\vec{r}(t) = \langle \cos t, 3\sin t, -t\rangle$, and show direction of increasing t. Give the equation of a surface on which this curve lies and show this surface on your sketch.

(12 points)

curve lies on surface

$$\left.\begin{aligned} x &= \cos t \\ y &= 3\sin t \end{aligned}\right\} \quad \begin{aligned} x &= \cos t \\ \frac{y}{3} &= \sin t \end{aligned} \Rightarrow x^2 + \frac{y^2}{9} = 1$$

$z = -t$



Coils equally spaced by 2π units